

Directed and Elliptic Flow

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We compare microscopic transport model calculations to recent data on the directed and elliptic flow of various hadrons in 2 – 10 A GeV Au+Au and Pb (158 A GeV) Pb collisions. For the Au+Au excitation function a transition from the squeeze-out to an in-plane enhanced emission is consistently described with mean field potentials corresponding to one incompressibility. For the Pb (158 A GeV) Pb system the elliptic flow prefers in-plane emission both for protons and pions, the directed flow of protons is opposite to that of the pions, which exhibit anti-flow. Strong directed transverse flow is present for protons and Λ 's in Au(6 A GeV)Au collisions as well. Both for the SPS and the AGS energies the agreement between data and calculations is remarkable.

Recently, it has been reported on an enormous amount of new detailed data on the collective flow in relativistic heavy ion collisions [1–3].

The excitation function of transverse collective flow is the earliest predicted signature for probing compressed nuclear matter [4]. Its sensitivity to the equation of state (EoS) can be used to search for abnormal matter states and phase transitions [5,6].

In the fluid dynamical approach, the transverse collective flow is directly linked to the pressure $P(\rho, S)$ (depending on the density ρ and the entropy S) of the matter in the reaction zone:

One can get a physical feeling for the generated collective transverse momentum \vec{p}_x by writing it as an integral of the pressure acting on a surface and over time [7]:

$$\vec{p}_x = \int_t \int_A P(\rho, S) dA dt. \quad (0.1)$$

Here dA represents the surface element between the participant and spectator matters and the total pressure is the sum of the potential pressure and the kinetic pressure: The transverse collective flow depends directly on the equation of state, $P(\rho, S)$.

Collective flow had originally been predicted by nuclear shock wave models and ideal fluid dynamics (NFD) [4,8–10]. Microscopic models such as VUU (Vlasov Uehling Uhlenbeck), and QMD (Quantum Molecular Dynamics) have predicted smaller flow than ideal NFD. These microscopic models agree roughly with viscous NFD [11] and with data [12–17], which discovered flow

first at the BEVALAC [18–20] for charged particles by the Plastic-Ball and Streamer Chamber collaborations [21], and at SATURNE by the DIOGENE collaboration [22,23]. It has been studied extensively at GSI by the FOPI [24,25], LAND [26], TAPS [27], and KaoS [28] collaborations, and by the EOS-TPC collaboration at LBNL [29] and at MSU [30].

Two different signatures of collective flow have been predicted:

- a) The *bounce-off* [8] of compressed matter *in the reaction plane* and
- b) the *squeeze-out* [9] of the participant matter *out of the reaction plane*.

The most strongly stopped, compressed matter around mid-rapidity is seen directly in the *squeeze-out* [16]. A strong dependence of these collective effects on the nuclear equation of state is predicted [16]. For higher beam energies, however, projectile and target spectator decouple quickly from the reaction zone, giving way to a preferential emission of matter in the reaction plane, even at mid-rapidity [31]. An excitation function of the *squeeze-out* at midrapidity shows the transition from out of plane enhancement to preferential in-plane emission.

At 10.6 A GeV collective flow has recently been discovered by the E877 collaboration [32,33] by measuring $dv_1/d\eta = d(\langle E_x \rangle / \langle E_T \rangle) / d\eta$ for different centrality bins. The EOS group has measured the flow excitation function for Au+Au at the AGS in the energy range between 2.0 and 8 GeV/nucleon [34]. Their data show a smooth decrease in $\langle p_x \rangle$ from 2 to 8 GeV/nucleon and are corroborated by measurements of the E917 collaboration at 8 and 10.6 GeV/nucleon [35].

The EOS collaboration has also measured a *squeeze-out* excitation function (sometimes also termed “elliptic flow” [31]), indicating a transition from out-of-plane to in-plane enhancement around 5 GeV/nucleon [34].

At CERN/SPS, the first observations of the predicted directed transverse flow component [36] have been reported by the WA98 collaboration [37,3] using the Plastic Ball detector located at target rapidity for event plane reconstruction. They show a strong directed flow signal for protons and “antiflow” for pions, both enhanced for particles with high transverse momenta. Similar findings have also been reported by the NA49 collaboration,

which due to their larger acceptance allows for a more detailed investigation [2].

Due to its direct dependence on the EoS, $P(\rho, T)$, flow excitation functions can provide unique information about phase transitions: The formation of abnormal nuclear matter, e.g., yields a reduction of the collective flow [5,17]. A directed flow excitation function as signature of the phase transition into the QGP has been proposed by several authors [5,6,38–42].

A microscopic analysis showed that the existence of a first order phase transition can show up as a reduction in the directed transverse flow [16,17].

For first order phase transitions, the pressure remains constant (for $T = \text{const}$) in the region of the phase coexistence. This results in vanishing shock velocities $v_f = 0$, $v_s = 0$ and velocity of sound $c_s = \sqrt{\partial p / \partial \varepsilon}$ [5,6].

The expansion of the system is driven by the pressure gradients, therefore expansion depends crucially on c_s^2 . Matter in the mixed phase expands less rapidly than a hadron gas or a QGP at the same energy density and entropy. In case of rapid changes in the EoS without phase transition, the pressure gradients are finite, but still smaller than for an ideal gas EoS, and therefore the system expands more slowly [43,44].

This reduction of c_s^2 in the transition region is commonly referred to as *softening* of the EoS. Here the flow will temporarily slow down (or possibly even stall). This hinders the deflection of spectator matter (the *bounce-off*) and, therefore, causes a reduction of the directed transverse flow [5,42,39,40] in semi-peripheral collisions. The softening of the EoS should be observable in the excitation function of the transverse directed flow of baryons.

An observation of the predicted local minimum in the excitation function of the directed transverse flow [5,40,45] would be an important discovery, and an unambiguous signal for a phase transition in dense matter. Its experimental measurement would serve as strong evidence for a QGP, if that phase transition is of first order.

An illustration of the in-plane elliptic flow is given by the following picture: Two colliding nuclei create a stopped overlap region. At higher bombarding energies ($E_{\text{lab}} \geq 10 \text{ A GeV}$) the spectators leave rapidly this interaction zone. The remaining interaction zone expands almost freely, where the surface is such that in-plane emission is preferred. It is therefore also the interplay between the timescales of passing time of the spectators and expansion time of the dense, stopped interaction zone which determines the time-integrated elliptic flow signal. Indeed, when following the elliptic flow as a function of reaction time, early out-of-plane squeeze is superposed by later preferential in-plane expansion [46]. So, the sign of the elliptic flow changes twice as a function of incident energy: At intermediate energies ($E_{\text{lab}} \approx 100 \text{ A MeV}$) a change from in-plane emission (rotation-like behaviour) to the squeeze-out is predicted [47,48] where at relativistic energies ($E_{\text{lab}} \approx 5 \text{ A GeV}$) the opposite change from the squeeze-out to in-plane enhancement is observed.

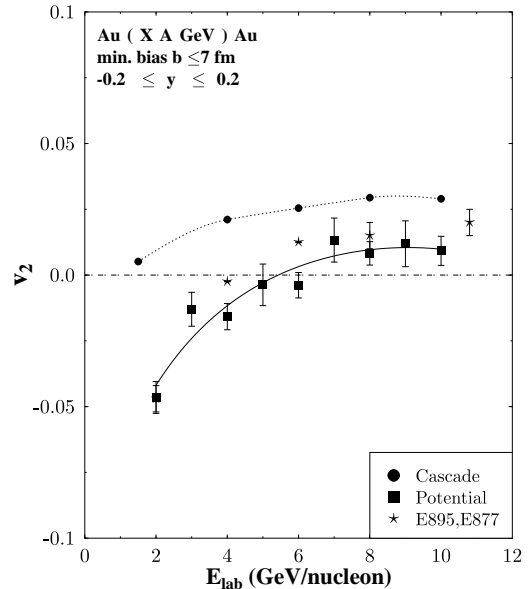


FIG. 1. Elliptical flow parameter v_2 as a function of incident energy E_{lab} for midrapidity protons in Au+Au collisions. Data of the E895 and E877 collaborations (stars) and UrQMD calculations with (squares) and without (circles) mean field potentials are displayed.

Fig. 1 shows the excitation function of the in-plane/squeeze-out flow parameter v_2 . This is observed by 90° peaks in the azimuthal angular distribution $dN/d\Phi$ [9,20,48,26,28] of nucleons at midrapidity for Au+Au collisions with the Fourier expansion

$$\frac{dN}{d\Phi} = v_0 (1 + 2 v_1 \cos(\Phi) + 2 v_2 \cos(2\Phi)) . \quad (0.2)$$

v_0 is for normalization only, where v_1 characterizes the directed in-plane flow. While $v_2 > 0$ indicates in-plane enhancement, $v_2 < 0$ characterizes the squeeze-out perpendicular to the event plane. Data by the E895 [1] and the E877 [49] collaborations (stars) and UrQMD calculations are displayed. The UrQMD calculations are performed within the cascade mode (circles) as well as with mean field potentials (squares). A detailed survey on the UrQMD model and its underlying concepts is available [50]. Clearly, the experimental observation of a transition from squeeze-out to a preferential in-plane emission can only be described with the potentials included. The cascade simulations do not show the squeeze-out due to the lack of the strongly repulsive nucleonic potential at this energy. The data are consistently described with potentials corresponding to an equation of state with *one* incompressibility ($K = 380 \text{ MeV}$), independent on the incident energy. This is in contrast to findings in [1] where a softening of the equation of state with incident energy is deduced from the comparison to transport model calculations [51].

Transverse flow has been discovered even at the highest energies at the SPS for the Pb+Pb system at 158 A GeV both by the NA49 and by the WA98 collaborations. Here,

UrQMD calculations are compared to the flow parameters v_1 and v_2 , which can also be expressed by [2]

$$v_1 = \langle \frac{p_x}{p_t} \rangle, \quad v_2 = \langle \left(\frac{p_x}{p_t} \right)^2 - \left(\frac{p_y}{p_t} \right)^2 \rangle. \quad (0.3)$$

Fig.2 shows the rapidity dependence of the proton flow (upper half) and of the flow of charged pions (lower half). Full symbols are UrQMD calculations where open symbols are experimental data [2]. The data are reflected at midrapidity ($y_{lab} \approx 2.9$). In reflection the signs of the v_1 values have been reversed in the backward hemisphere, but not the v_2 values [2]. For the directed transverse flow (v_1), both data as well as UrQMD results exhibit a characteristic S-shaped curve. The elliptic flow values (v_2) seem to be slightly peaked at medium rapidity ($y_{lab} \approx 4-4.5$ and $y_{lab} \approx 1.5-2$), both for pions and protons, contrary to what was inferred in [2]. Both protons and pions show an in-plane enhanced emission ($v_2 > 0$).

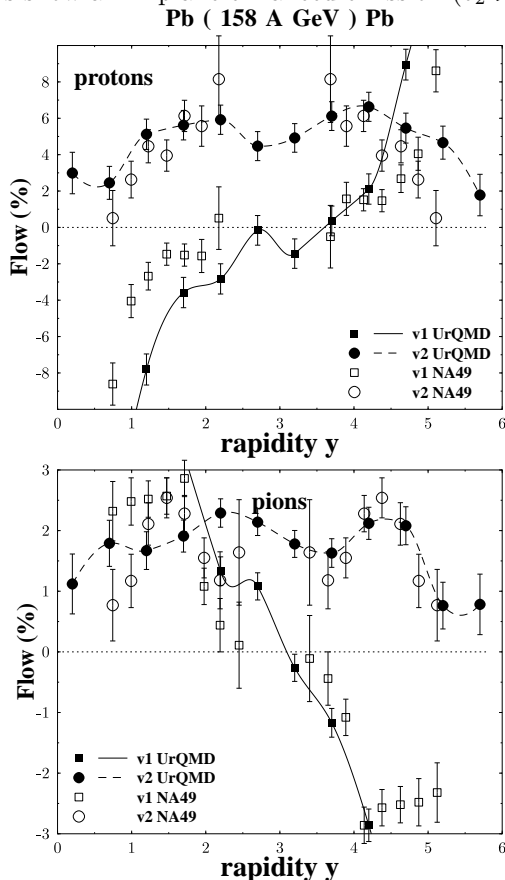


FIG. 2. Flow parameters v_1 and v_2 as a function of rapidity for protons (upper diagram) and charged pions (lower diagram). Open symbols are data and full symbols display the UrQMD results

The proton flow shows positiv flow whereas the pion flow exhibits the opposite negative sign, caused by absorption and rescattering effects. The overall agreement between data and calculations looks rather good. Discrepancies are seen dominantly for the high rapidity pion

directed flow (v_1), which is too strong in the calculations compared to the data which show saturation of v_1 for $y_{lab} > 4$ and $y_{lab} < 2$. Also the proton directed flow seems to be slightly too strong at high rapidity. The elliptic flow shows good agreement for the sign as well as for the magnitude of v_2 ($v_2 \approx 5\%$ for protons and $v_2 \approx 2\%$ for pions).

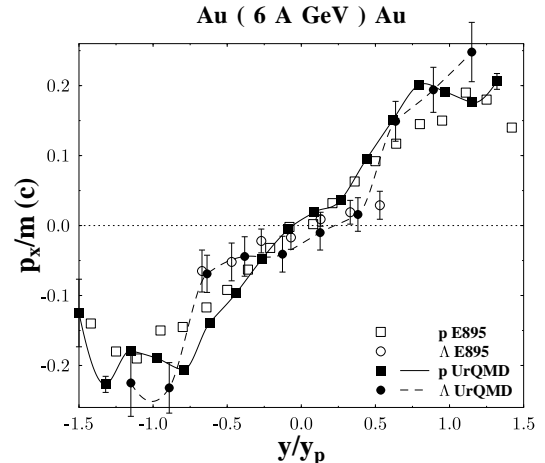


FIG. 3. Directed transverse flow p_x/m as a function of normalized rapidity y/y_p for protons (squares) and Λ 's (circles). Open symbols are data and full symbols are the results of UrQMD calculations.

Strong directed flow has also been discovered in the energy region where the elliptic flow disappears. Fig. 3 shows the directed transverse flow p_x/m as a function of the normalized rapidity for protons (squares) and Λ 's (circles) in Au(6 A GeV)Au collisions. Open symbols are preliminary data by the E895 collaboration [1] and full symbols display the results of UrQMD calculations. The proton data are reflected at midrapidity. Both, protons and Λ 's show strong positive directed flow. The proton flow is larger than the Λ flow close to midrapidity ($|y/y_p| \leq 0.6$) both in the data as well as in the UrQMD calculations. At target/projectile rapidity the Λ flow is predicted to exhibit a similar magnitude as the protons show. The species-dependent flow pattern clearly demonstrates a complex non-hydrodynamic behaviour which seems to rule out simple fireball+flow models.

In summary, recent data on the collective flow in heavy ion collisions at the SPS and AGS have been compared to UrQMD calculations. The excitation function of the elliptic flow at midrapidity for the Au+Au system shows a transition from the squeeze-out to an in-plane enhancement. The data agree with the calculations done with an equation of state with *one* incompressibility. Therefore a softening of the equation of state cannot be deduced from this comparison.

The elliptic flow at the SPS for Pb+Pb collisions shows in-plane enhancement, both for protons and pions in the full rapidity range. The UrQMD results show complete agreement to data. The positive directed flow of protons

is opposite to the directed flow of pions which show an anti-flow. While good agreement exists around midrapidity, the pion flow is too strong in the calculations at high rapidities. This seems to be due to the high momentum tails of the pion transverse momentum distribution and will be investigated in a forthcoming publication. The directed proton flow also seems to be slightly overestimated at high rapidities by the UrQMD results.

The comparison of Au(6 A GeV)Au collisions demonstrates that strong directed flow is present for protons and Λ 's, where the Λ 's show less flow than the protons around midrapidity. At higher rapidities the Λ flow is predicted to show similar magnitude as the proton flow.

The species-dependent flow patterns illustrate the complex collision dynamics and demonstrate the necessity of highly non-trivial microscopic transport models for an adequate description of relativistic heavy ion collisions.

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